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A CORRELATION OF PSYCHROMETRIC RATIOS FOR A FLAT PLATE

J. Y. KAUH,† R. E. PECK and D. T. WASAN
Department of Chemical Engineering, Illinois Institute of Technology, Chicago, Illinois

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NOMENCLATURE

C_p, heat capacity;
D, molecular diffusivity;
h_c, convective heat-transfer coefficient;
k_c, heat conductivity;
k_g, convective mass-transfer coefficient;
L, length of a slab;
M_m, mean molecular weight;
Nu_h, Nusselt number for heat transfer, h_cL/k_c;
Nu_m, Sherwood number for mass transfer, k_gL/D;
Pr, Prandtl number, $C_p\mu/k_c$;
Re, Reynolds number, u_sL/v;
Sc, Schmidt number, v/D;
St_h, Stanton number for heat transfer, $Nu_h/Pr\,Re$;
St_m, Stanton number for mass transfer, $Nu_m/Sc\,Re$;
u, velocity in the axial direction;
u_s, free stream velocity;
V, velocity in the normal direction;
μ, molecular viscosity;
ρ, density;
B, psychrometric ratio defined by equation (1);
v, kinematic viscosity;
P_{bm}, log mean inert partial pressure.

† J. Y. Kauh is presently employed with the Union Carbide Corporation, S. Charleston, West Virginia.

INTRODUCTION

THE ANALOGY between momentum, heat and mass transfer has long been used for the correlation of heat- and mass-transfer coefficients for turbulent flow in pipes. The empirical extension of the methods used for pipes has been successfully employed for different shapes [1, 2, 3]. These correlations have steadily improved as more accurate knowledge of velocity and eddy viscosity distribution has been obtained.

The analogy between momentum, heat and mass transfer in turbulent flow has been used by several investigators including Bedingfield and Drew [4] and Lynch and Wilke [5, 6] for analyzing their psychrometric data. Even though much effort has been directed towards predicting the psychrometric ratio [7], no completely satisfactory agreement has been found among the numerous empirical relations which have been proposed. Wilke and Wasan [8] proposed a correlation for the psychrometric ratio based on their recent analysis of the transfer of heat and mass in pipe flow [9, 10].

The application of analogy for the prediction of the psychrometric ratio for a given geometry, should be possible if the necessary correlations of velocity distribution and friction coefficients for gas flow have been developed. Most of the existing psychrometric data have been obtained for a thermometer bulb of an approximately cylindrical shape. Wilke and Wasan made an attempt to apply the general form of the analogy for mass and heat transfer in pipes to the wet bulb problem. The assumption was made that the velocity distribution outside a single cylinder can be represented by the distribution for the flow in the vicinity of a cylindrical pipe. This assumption is reasonable for a cylinder with moderate curvature. These authors [8] proposed the following simplified expression for the psychrometric ratio:

$$B = \frac{kgP_{bm}M_mC_p}{h_c} = \frac{1 + 0{\cdot}7(Pr^{0{\cdot}77} - 1)}{1 + 0{\cdot}7(Sc^{0{\cdot}77} - 1)}. \quad (1)$$

Equation (1) was found to represent the psychrometric ratio data well at very high gas velocities for a Reynolds number of 10^5.

In the present communication we are concerned with the prediction of a correlation for psychrometric ratios for a flat plate. The theoretical psychrometric ratio for a flat plate has been compared with the case of a cylinder.

PSYCHROMETRIC RATIO FOR A FLAT PLATE

Heat transport in turbulent flow near the surface of a flat plate has been studied by numerous investigators. Rubesin [11], Furber [12], Scesa and Sauer [13], Owen and Ormerod [14], Reynolds, Kays and Kline [15] presented the correlations of experimental data for the cases where the Prandtl number is near unity. Spalding [16] presented a new approach to the more general problem. He also proposed a new form of the law of the wall [17] in which a single equation is used to describe the dimensionless velocity variation in the boundary layer. Kestin and Persen [18], Smith and Shah [19] and Gardner and Kestin [20] have extended Spalding's work and produced a solution to the problem for a wide range of Prandtl numbers.

Kestin and Persen [18] calculated the values of the Spalding function numerically for a Prandtl number of unity. The Spalding function is defined as follows:

$$Sp(x^+, Pr) = -\left(\frac{\partial \theta}{\partial u^+}\right)_{u^+=0} \quad (2)$$

where

$$\theta = \frac{T - T_g}{T_s - T_g}, \quad (3)$$

$$u^+ = u/v^*, \quad (4)$$

$$v^* = (\tau_w/\rho)^{\frac{1}{2}} \quad (5)$$

and

$$x^+ = \int_0^x \frac{v^*}{\nu}\,dx. \quad (6)$$

Gardner and Kestin (20) extended the calculation and obtained the values of Spalding's function for Prandtl numbers which are different from unity. They presented the result as the function of x^+ and Pr. The local Stanton number is written as

$$St_h = Sp\,.\,(1/Pr)\,.\,(C_f/2)^{\frac{1}{2}} \quad (7)$$

where C_f is local coefficient of friction.

In actual application, the overall Stanton number is based on the entire area of transfer rather than local Stanton number. The overall Stanton number for a given length L of a flat plate can be written as

$$\overline{St_h} = \frac{1}{L}\int_0^L Sp(1/Pr)\,(C_f/2)^{\frac{1}{2}}\,dx; \quad (8)$$

or

$$\overline{St_h} = \frac{1}{Re}\int_0^{Re} Sp(1/Pr)\,(C_f/2)^{\frac{1}{2}}\,dRe; \quad (9)$$

where

$$Re = (u_s x)/\nu$$

and

$$Re = (u_s L)/\nu.$$

To integrate equation (9), it is necessary to have Sp and C_f in terms of Re rather than x^+. From the definition of x^+ it is possible to relate x^+ to Re:

$$x^+ = \int_0^x (v^*/\nu)\,dx = \int_0^{Re} (1/u_s^+) \int_0^{u_s^+} (u^+)^2 \left(\frac{dy^+}{du^+}\right) du^+\,du_s^+. \quad (10)$$

Spalding gives a form of the law of the wall which is a single equation:

$$y^+ = u^+ + 0{\cdot}1108 \left\{\exp(0{\cdot}4\,u^+) - 1 - 0{\cdot}4\,u^+ - \frac{(0{\cdot}4\,u^+)^2}{2!} - \frac{(0{\cdot}4\,u^+)^3}{3!} - \frac{(0{\cdot}4\,u^+)^4}{4!}\right\} \quad (11)$$

where

$$y^+ = y(v^*/\nu). \quad (12)$$

From the equation of the law of the wall it follows that

$$x^+ = \frac{(u_s^+)^3}{9} + 0{\cdot}1108\left(\frac{1}{0{\cdot}4}\right)^2 \left\{\exp(0{\cdot}4\,u_s^+)(0{\cdot}4\,u_s^+ - 3) + \sum_{i=1}^{6} (3 - i)\frac{(0{\cdot}4\,u_s^+)^i}{i!} + 2\sum_{i=7}\frac{1}{i}\frac{(0{\cdot}4\,u_s^+)}{i!} + 3\right\}. \quad (13)$$

The relation between u_s^+ and Re can be found as follows:

$$Re = \int_0^{u_s^+}\int_0^{u_s^+} (u_s^+)^2 \left(\frac{dy^+}{du^+}\right) du_s^+ =$$

$$= \frac{(u_s^+)^4}{12} + 0.1108\left(\frac{1}{0.4}\right)^3\Bigg[\exp(0.4\,u_s^+)\{(0.4\,u_s^+)^2 - 4(0.4\,u_s^+) + 6\} - \frac{2}{4!}(0.4\,u_s^+)^4 - \frac{6}{5!}(0.4\,u_s^+)^5 - \frac{12}{6!}(0.4\,u_s^+)^6 - \frac{20}{7!}(0.4\,u_s^+)^7 - 2(0.4\,u_s^+) - 6\Bigg]. \tag{14}$$

Since

$$(C_f/2)^{\frac{1}{2}} = (\tau_w/\rho\,u_s^2)^{\frac{1}{2}} = 1/u^+, \tag{15}$$

both the overall and local Stanton numbers are calculated for the variable Re with the parameter Pr.

Sample calculations are given for Prandtl numbers of 0·71, 1·0, 7·0 and for the Reynolds number range of 10^2–10^6. The results are shown in Fig. 1.

By applying analogy to the calculation of the mass- and heat-transfer coefficients, the psychrometric ratio is written as follows:

$$B = \frac{\overline{St}_m}{\overline{St}_h}. \tag{16}$$

where

$$\overline{St}_m = \left(\frac{1}{Re\,Sc}\right)\int_0^{Re}\left(\frac{Sp(Re', Sc)}{u_s^+}\right)\mathrm{d}Re' \tag{17}$$

and

$$\overline{St}_h = \left(\frac{1}{Re\,Pr}\right)\int_0^{Re}\left(\frac{Sp(Re', Pr)}{u_s^+}\right)\mathrm{d}Re'. \tag{18}$$

DISCUSSION

Equation (1) which represents the wet-bulb thermometry data well suggests that the psychrometric ratio should be a single function of the ratio of the Schmidt to the Prandtl number at any given Reynolds number. Furthermore, most experimental studies have not revealed any appreciable variation of the psychrometric ratio with high gas velocities. Accordingly, the theoretical psychrometric ratio for a flat plate is calculated using equations (16–18) at $Re = 10^4$ and $Re = 10^5$ as shown in Fig. 2. These Reynolds numbers were chosen because they embrace the range of most data for wet bulb thermometry.

The flow characteristics past a flat plate are different from those around a single cylinder. However, in the past the similarity between the flow fields in the vicinity of a pipe wall and around the cylinder lead to rather good prediction of heat- and mass-transfer rates [2, 8]. Therefore, the psychrometric ratio for flat plates are compared with those of single cylinders in Fig. 2. It is to be noted that there are large differences between curves (a) and (b) which are based on $Re = 10^4$. But there is hardly any difference at $Re = 10^5$ between the curve (c) which is based on equation (16) for a flat plate and the curve (d) which is calculated from equation (1) for a cylinder over a wide range of the ratio of Schmidt to Prandtl number. Therefore, equation (1) may well be used to predict the psychrometric ratio at very high gas velocities in the absence of experimental information.

Recently, two of these authors [21, 22] employed the

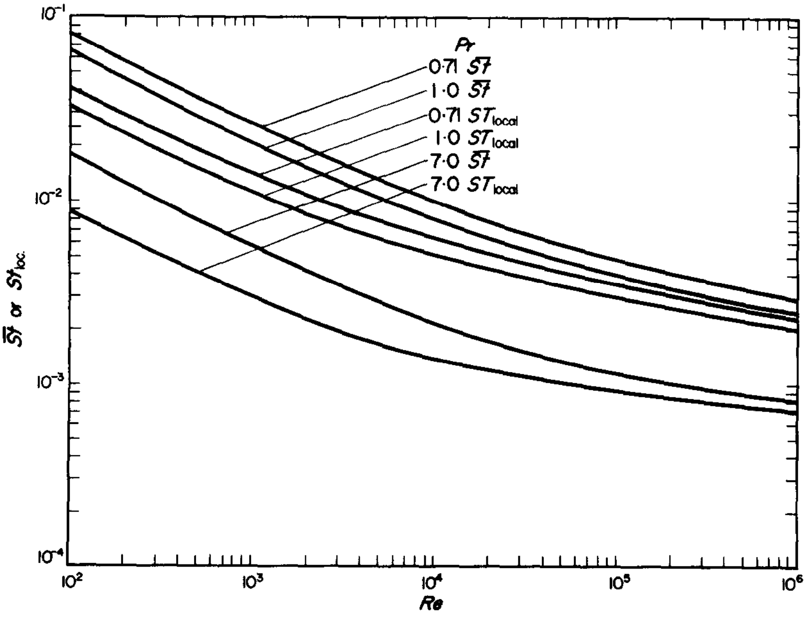

FIG. 1. A correlation of Stanton number for a flat plate.

present equation (16) in the computation of drying schedules for drying of slabs of balsa wood with air and found a good agreement between the predicted and observed values.

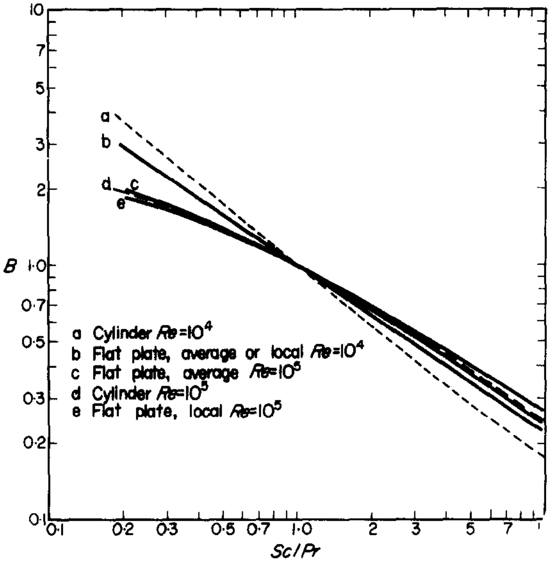

FIG. 2. A correlation of psychrometric ratio for a flat plate.

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